25 Years

Previous Years Solved Papers

Indian Forest Service Main Examination

(2000-2024)

Civil Engineering

Paper-I

Topicwise Presentation

Also useful for Engineering Services Main Examination,
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Civil Engineering : Indian Forest Service Main Examination (Paper-I)

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Preface

Our country has a vast forest cover of near about 25% of geographical area and if man doesn't learn to treat trees with respect, man will become extinct; Death of forest is end of our life. Scientific management and judicial exploitation of forest becomes first task for sustainable development.

Engineer is one such profession which has an inbuilt word "Engineer – skillful arrangement" and hence IFS is one of the most talked about jobs among engineers to contribute their knowledge and skills for the arrangement and management for sustainable development

In order to reach to the estimable position of Divisional Forest Officer (DFO), one needs to take an arduous journey of Indian Forest Service Examination. Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make Indian Forest Service Examination plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years' papers of Indian Forest Service Examination. The book aims to provide complete solution to all previous years' questions with accuracy.

On doing a detailed analysis of previous years' Indian Forest Service Examination question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Services and State Services exams. Hence, this book is a one stop shop for all Indian Forest Service Examination, CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years' papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes

B. Singh

CMD, MADE EASY Group

Previous Years Solved Papers

Indian Forest Service Main Examination

Civil Engineering

Paper-I

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Part-A

ENGINEERING MECHANICS, STRENGTH OF MATERIALS AND STRUCTURAL ANALYSIS

ENGINEERING MECHANICS :

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Kinematics and Kinetics : *Kinematics in Cartesian and Polar Coordinates, motion under uniform and non-uniform acceleration, motion under gravity. Kinetics of particle : Momentum and Energy principles, D' Alembert's Principle, Collision of elastic bodies, rotation of rigid bodies, simple harmonic motion, Flywheel.*

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Dimensional Analysis and Similitude: *Buckingham's Pi-theorem, dimensionless parameters, similitude theory, model laws, undistorted and distorted models.*

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■■■■

1 Strength of Materials

1. Properties of Metal and Basic Concepts

1.1 The bars AB, AC and AD shown in indeterminate system (see figure below) are made of steel and have same cross-sectional area of 350 mm^2 and they together carry a load of 75 kN, applied at A as shown. There is no initial stress in bars before application of load. α = 30° and l = 3000 mm. Find force in each bar and vertical displacement of point A after load is applied. Take E = 205 kN/mm^2.

[15 marks : 2002]

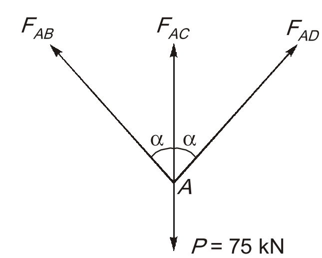
Solution:

Let the force in AB, AC and AD are F_{AB}, F_{AC} and F_{AD}

$$\alpha = 30°$$

$$P = 75 \text{ kN}, L_{AC} = L = 3 \text{ m}$$

In equilibrium condition



$$\Sigma F_x = 0$$

$$\Rightarrow \quad F_{AB} \sin 30° - F_{AD} \sin 30° = 0$$

$$\Rightarrow \quad F_{AB} = F_{AD} \qquad \text{...(i)}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad F_{AB} \cos 30° + F_{AC} + F_{AD} \cos 30° = P$$

$$\Rightarrow \quad 2F_{AB} \times \frac{\sqrt{3}}{2} + F_{AC} = P$$

$$\Rightarrow \quad F_{AC} = \left(P - \sqrt{3}F_{AB}\right) \qquad \text{...(ii)}$$

Total strain energy

$$U = V_{AB} + V_{AC} + V_{AD} = \frac{F_{AB}^2 L_{AB}}{2AE} + \frac{F_{AC}^2 L_{AC}}{2AE} + \frac{F_{AD}^2 L_{AD}}{2AE}$$

$$\Rightarrow \quad U = \left\{\frac{F_{AB}^2 \times 2L}{2AE \times \sqrt{3}} \times 2\right\} + \frac{\left(P - \sqrt{3}F_{AB}\right)^2 L}{2AE}$$

for minimum strain energy

$$\frac{\partial U}{\partial F_{AB}} = 0$$

$$\Rightarrow \quad \frac{L}{2AE}\left(\frac{4}{\sqrt{3}} \times 2F_A + 2\left(P - \sqrt{3}F_{AB}\right) \times -\sqrt{3}\right) = 0$$

$$\Rightarrow \quad F_{AB} = \frac{3P}{4 + 3\sqrt{3}} = \frac{3 \times 75}{4 + 3\sqrt{3}} = 24.47 \text{ kN}$$

$$F_{AD} = F_{AB} = 24.47 \text{ kN}$$

$$F_{AC} = P - \sqrt{3}F_A = 75 - \sqrt{3} \times 24.47 = 32.62 \text{ kN}$$

Vertical detection of point A (from Castigliano's Ist theorem)

$$\Delta V_A = \frac{\partial V}{\partial P} = \frac{\partial}{\partial P}\left\{\frac{2F_{AB}^2 \times 2L}{2AE \times \sqrt{3}} + \frac{\left(P - \sqrt{3}F_{AB}\right)^2 L}{2AE}\right\}$$

$$= \frac{L}{2AE}\left\{\frac{L}{\sqrt{3}} \times 2F_{AB} \times \frac{\partial F_{AB}}{\partial P} + 2\left(P - \sqrt{3}F_{AB}\right)L\left(1 - \sqrt{3}\frac{\partial F_{AB}}{\partial P}\right)\right\}$$

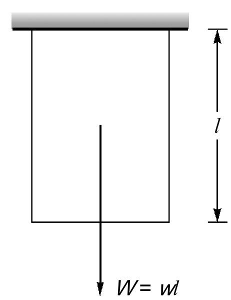
$$= \frac{1}{2AE}\left\{\frac{4L}{\sqrt{3}} \times 2F_{AB} \times \left(\frac{3}{4+3\sqrt{3}}\right) + 2\left(P - \sqrt{3}F_{AB}\right)L\left(1 - \sqrt{3} \times \frac{3}{4+3\sqrt{3}}\right)\right\}$$

$$= \frac{1}{(2 \times 350 \times 10^{-6} \times 205 \times 10^{6})}\left\{\frac{4 \times 3}{\sqrt{3}} \times 2 \times 24.47 \times \frac{3}{4+3\sqrt{3}}\right\}$$

$$+2 \times \left(75 - \sqrt{3} \times 24.47\right) \times 3\left(1 - \frac{3\sqrt{3}}{4+3\sqrt{3}}\right)$$

$$= \frac{195.72 \times 1}{2 \times 305 \times 205} = 1.36 \times 10^{-3}\,\text{m} = 1.36\text{ mm}$$

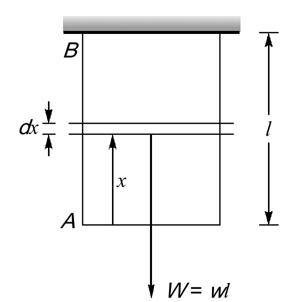
1.2 Find elongation of a bar of uniform cross-section area '*A*' and length '*l*' under action of its own weight. The bar weight '*w*' per unit length. E = Modulus of elasticity. See figure below:



[10 marks : 2002]

Solution:

Take element of cross-sectional area '*A*' with length '*dx*' elongate under weight w'



$$w' = wx$$

Δl = Total elongation, dl = Elements elongation

$$dl = \frac{(wx)dx}{AE}$$

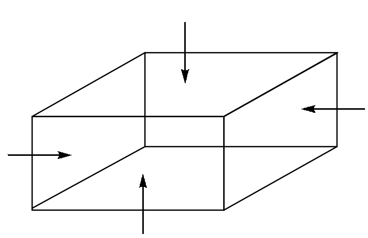
$$\int dl = \Delta l = \int_0^l \frac{(Wx)dx}{AE}$$

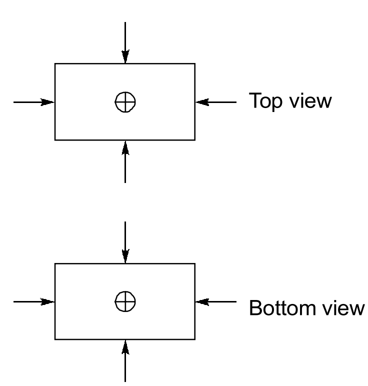
$$\Delta l = \frac{wl^2}{2AE} = \frac{Wl}{2AE}$$

1.3 A 50 mm cube is subjected to uniform pressure of 200 MPa. When the change in dimension between 2 parallel faces of cube is 0.025 mm. Determine change in volume of cube, μ = 0.25.

[8 marks : 2010]

Solution:





$$p = 200 \text{ MPa}$$

$$\Delta l = \left[-\frac{p}{E} + \mu \times \frac{p}{E} + \mu \frac{p}{E}\right] \times l$$

Given, $\Delta l = -0.025$ mm, $l = 50$ mm, p = 200 MPa

$$\frac{50 \times 200}{E} \times (2 \times 0.25 - 1) = -0.025$$

$$E = 2 \times 10^5 \text{ MPa}$$

$$B = \text{Bulk modulus} = \frac{E}{3(1-2\mu)}$$

$$B = \frac{2 \times 10^5}{3 \times (1 - 2 \times 0.25)} = 1.33 \times 10^5 \text{ MPa}$$

$$B = \frac{-p}{(\Delta V / V)}$$ (p should be insulated with sign)

$$1.33 \times 10^5 = \frac{-(-200)}{\Delta V / (50)^3}$$

$$\frac{\Delta V}{(50)^3} = 150 \times 10^{-5}$$

$$\mathbf{\Delta V = 187.5 \text{ mm}^3}$$

Alternative

$$\Delta l_1 = 0.025 \text{ mm}$$

$$\frac{\Delta V}{V} = \epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3$$

As it is cube so $\Delta l_1 = \Delta l_2 = \Delta l_3$ (Uniform pressure)

$$\epsilon_V = \left(\frac{0.025}{50}\right) \times 3 = 1.5 \times 10^{-3}$$

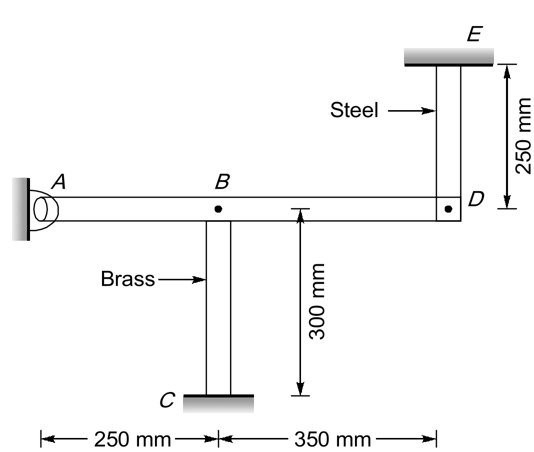
$$\frac{\Delta V}{V} = 1.5 \times 10^{-3}$$

$\Rightarrow$ $$\Delta V = (1.5 \times 10^{-3})\, V$$

$$\Delta V = 187.5 \text{ mm}^3$$

1.4 A rigid bar *AD* is pinned at *A* and attached to the bars *BC* and *ED* as shown in figure. The entire system in initially stress free and weights of all bars are negligible. The temperature of bar BC is lowered 25°C and of bar *ED* is raised 25°C. Neglecting any possibility of lateral buckling, find normal stress in bars BC and *ED*. For BC which is brass, assume E = 90 GPa, $\alpha = 20 \times 10^{-6}$/°C and for *ED*, which is steel take E = 200 GPa and $\alpha = 12 \times 10^{-6}$/°C. Cross-sectional area of *BC* is 500 mm² and of *ED* is 250 mm².

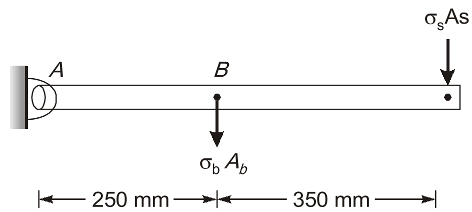
[10 marks : 2011]



Solution:

Let stress in brass $= \sigma_b$ (Tensile)

Stress in steel $= \sigma_s$ (Compressive) (Assume)

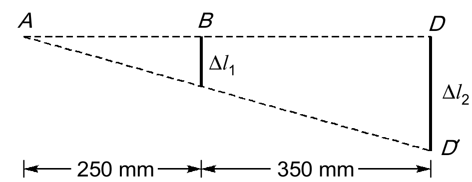


$$(\Sigma M)_A = 0 \quad \text{for static rotational equilibrium}$$

$$\sigma_b A_b \times 250 + \sigma_s A_s \times 600 = 0$$

$$\sigma_b \times 500 \times 250 + \sigma_s \times 250 \times 600 = 0$$

$$\sigma_b = (-\sigma_s)\, 1.2 \qquad \ldots\text{(i)}$$



Now as *ABD* bar is rigid so

$$\frac{\Delta l_1}{250} = \frac{\Delta l_2}{600}$$

$$\Delta l_2 = 2.4\Delta l_1$$

$$\Delta l_1 = 300 \times 20 \times 10^{-6} \times 25 - \frac{\sigma_b}{0.9 \times 10^5} \times 300$$

$$\Delta l_1 = 0.15 - 3.33 \times 10^{-3}\, \sigma_b \text{ (mm)}$$

All stress values are in N/mm^2

$$E = \text{in N/mm}^2 \text{ or MPa}$$

$$\Delta l_2 = l_s \alpha_s \Delta T - \frac{\sigma_s}{E_s} l_s$$

$$\Delta l_2 = 250 \times 12 \times 10^{-6} \times 25 - \frac{\sigma_s}{2 \times 10^5} \times 250$$

$$\Delta l_2 = 0.075 - 1.25 \times 10^{-3}\, \sigma_s$$

Now, $\quad 0.075 - 1.25 \times 10^{-3}\, \sigma_s = 2.4\,(0.15 - 3.33 \times 10^{-3}\, \sigma_b)$

$\Rightarrow \quad 8 \times 10^{-3}\, \sigma_b - 1.25 \times 10^{-3}\, \sigma_s = 0.285$

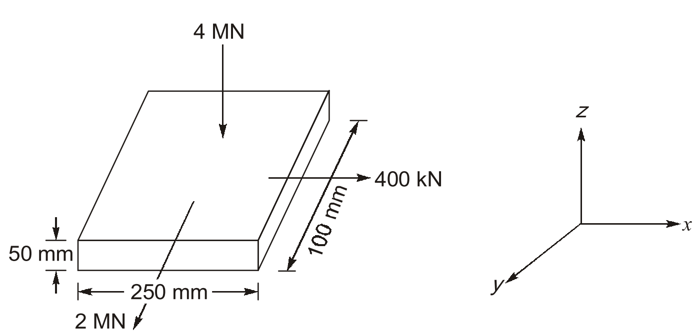
$$8\, \sigma_b - 1.25\, \sigma_s = 285 \qquad \ldots\text{(ii)}$$

$$\sigma_b = 31.521 \text{ N/mm}^2 \quad \text{(Tensile)}$$

$$\sigma_s = -26.27 \text{ N/mm}^2$$

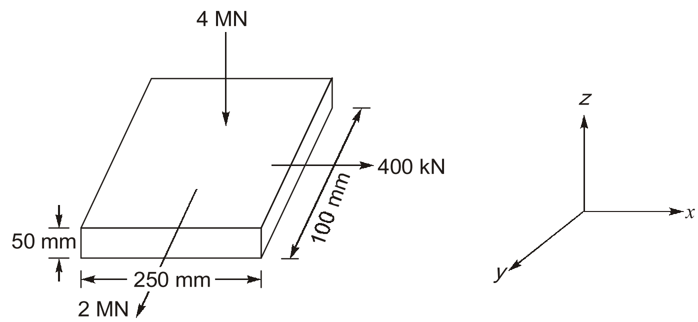
$$\sigma_s = 26.27 \text{ N/mm}^2 \text{ (Tensile)}$$

1.5 A metallic bar 250 mm× 100 mm × 50 mm is loaded as shown in below figure. Workout change in volume. What should be change that should be made in 4 MN load in order that there should be no change in the volume of the bar? Assume $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.25$.



[15 marks : 2012]

Solution:



$$\text{Stress in } x\text{-direction, } \sigma_x = \frac{2\times10^6}{250\times50} = 160 \text{ N/mm}^2 \qquad \left\{\sigma = \frac{P}{A}\right\}$$

$$\text{Stress in } y\text{-direction, } \sigma_y = \frac{400\times10^3}{100\times50} = 80 \text{ N/mm}^2$$

$$\text{Stress in } z\text{-direction, } \sigma_z = \frac{-4\times10^6}{100\times250} = -160 \text{ N/mm}^2$$

$$\text{Strain in } x\text{-direction, } \epsilon_x = \frac{\sigma_x}{E} - \frac{\mu(\sigma_y+\sigma_z)}{E}$$

$$\text{Strain in } y\text{-direction, } \epsilon_y = \frac{\sigma_y}{E} - \frac{\mu(\sigma_x+\sigma_z)}{E}$$

$$\text{Strain in } z\text{-direction, } \epsilon_z = \frac{\sigma_z}{E} - \frac{\mu(\sigma_x+\sigma_z)}{E}$$

$\therefore$ Volumetric strain,

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z = \frac{(\sigma_x+\sigma_y+\sigma_E)(1-2\mu)}{E}$$

$$\Rightarrow \quad \epsilon_V = \frac{(160+80-160)}{2\times10^5} = 4\times10^{-4}$$

$\therefore$ Change in volume, $\Delta v = \epsilon_v \cdot V = 4\times10^{-4}\times(100\times250\times50)$

$= 500 \text{ mm}^2$

Let the new stress in E-direction σ'_E so that volume charge is zero.

$$\Delta_V = \frac{(\sigma_x+\sigma_y+\sigma'_z)}{E}(1-2\mu)\cdot V = 0$$

$$\Rightarrow \quad \sigma_x+\sigma_y+\sigma'_z = 0$$

$$\Rightarrow \quad \sigma'_z = -(\sigma_x+\sigma_y) = -(160+80) = -240 \text{ N/mm}^2$$

$$P'_z = -240\times100\times250 = -6\times10^6 \text{ N} = -6 \text{ MN}$$

$\therefore$ Change in load in z-direction, $\Delta P_z = -6-(-4) = -2$ MN

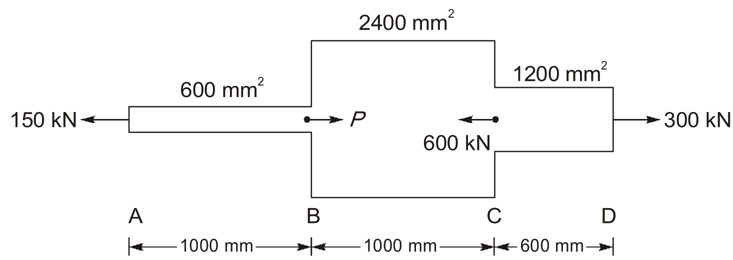
$= 2$ MN (compressive)

1.6 $E = 2 \times 10^5$ N/mm^2

A member *ABCD* is subjected to concentrated loads as shown. Calculate

(i) Force *P* necessary for equilibrium

(ii) Total elongation of bar



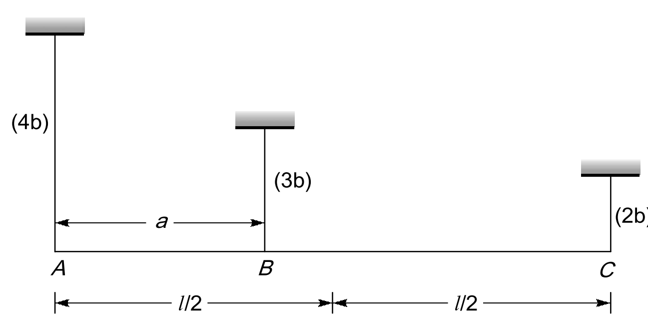
[8 marks : 2015]

Solution:

(i)
$$\Sigma F = 0$$
$$(P + 300) - (150 + 600) = 0$$
$$P = 450 \text{ kN}$$

(ii)
$$\Delta_{Total} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$
$$= \frac{150 \times 1000 \times 1000}{600 \times 2 \times 10^5} - \frac{300 \times 1000 \times 1000}{2400 \times 2 \times 10^5} + \frac{300 \times 600 \times 1000}{1200 \times 2 \times 10^5}$$
$$= 1.25 - 0.625 + 0.75$$
$$= 1.375 \text{ mm (elongation)}$$

1.7 Three steel bars *A*, *B* and *C* having same axial rigidity *AE* support a horizontal rigid beam *A*, *B*, *C* as shown in figure. Determine distance 'a' between bars *A* and *B*. In order that rigid beam will remain horizontal. When a load '*F*' is applied at its mid point. The value of length is given within parenthesis.



[8 marks : 2015]

Solution:

The beam is rigid and as per given condition/situation of beam to be remain horizontal following conditions must be satisfied.

1. Static equation
2. Axial elongation in all steel bars is equal and it takes place gradually.
3. There should be no net moment about any point.

From left, strings are named as 1, 2, and 3 respectively.

$$F_1 + F_2 + F_3 = F \quad \text{...(i)}$$
$$\frac{F_1(4b)}{AE} = \frac{F_2(3b)}{AE} = \frac{F_3(2b)}{AE} \quad \text{...(ii)}$$
$$4F_1 = 3F_2 = 2F_3 \quad \text{...(iii)}$$

$$\Rightarrow \quad F_1 + \frac{4F_1}{3} + \frac{4}{2}F_1 = F$$

$$\Rightarrow \quad \frac{13}{3}F_1 = F$$

$$\Rightarrow \quad F_1 = \frac{3}{13}F$$

$$\Rightarrow \quad F_2 = \frac{4}{13}F$$

$$\Rightarrow \quad F_3 = \frac{6F}{13}$$

Now, $(\Sigma M)_A = 0$ to prevent any rigid body rotation.

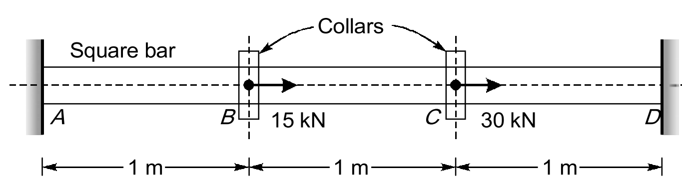
$$F_2(a) + F_3(l) = F\left(\frac{l}{2}\right)$$

$$\frac{4F}{13}a + \frac{6F}{13}l = \frac{Fl}{2}$$

$$\frac{4F}{13}a = Fl \times \frac{1}{26}$$

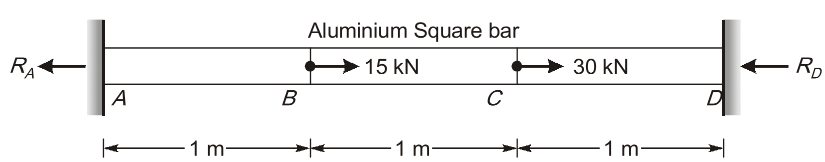
$$a = \frac{l}{8}$$

1.8 An aluminium square bar having the cross-section 50 mm × 50 mm and length 3 metres is fixed between two rigid supports as shown in the figure. Two loads, 15 kN and 30 kN are applied concentrically to the rod through collars as shown. Determine the stress developed at the right end of the bar. Young's modulus of aluminium is 70×10^9 N/m^2.



[10 marks : 2016]

Solution:

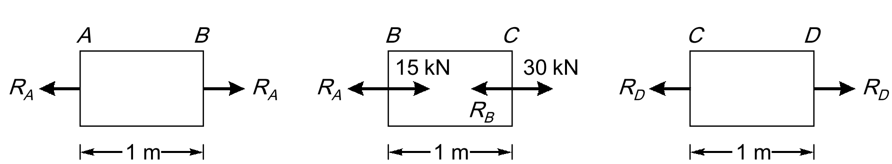


Given, Cross-section of aluminium bar = 50 mm × 50 mm

Young's no duels of aluminium, $E = 70 \times 10^9$ N/m^2

Let the reactions at A and D are R_A and R_D respectively.

Drawing free body diagrams individually;



Let there is tension in CD, let it be x, then

$$x = R_A - 15 = 30 - R_D$$

$$\Rightarrow \quad R_A + R_D = 45 \text{ kN} \qquad \text{...(i)}$$

$$\Delta_{AB} = \frac{R_A \times 1000}{AE} \text{ (Elongation)} \quad (\because \text{ Bar is prismatic, } AE \text{ is constant})$$

$$\Delta_{BC} = \frac{x \times 1000}{AE} \text{(Elongation)}$$

$$\Delta_{CD} = \frac{R_D \times 1000}{AE} \text{(Compression)}$$

But A & D are fixed, $\Delta_{AB} + \Delta_{BC} + \Delta_{CD} = 0$

$$\frac{R_A \times 1000}{AE} + \frac{x \times 1000}{AE} - \frac{R_D \times 1000}{AE} = 0$$

$$R_A + x - R_D = 0$$

$$\Rightarrow \quad R_A + (R_A - 15) - R_D = 0$$

$$\Rightarrow \quad 2R_A - 15 - R_D = 0$$

$$\Rightarrow \quad 2R_A - R_D = 15 \qquad \text{...(ii)}$$

Solving equation (i) and (ii), we get $R_A = 20$ kN

$$R_D = 25 \text{ kN}$$

$\therefore$ Stress developed on right end of bar = Stress at fixed end D

$$\sigma_0 = \frac{-R_D}{A} = \left(\frac{-25000}{50 \times 50}\right) = -10 \text{ N/mm}^2 \text{ (Compression)}$$

1.9 If two pieces of materials 'A' and 'B' have the same bulk modulus, but the value of Modulus of Elasticity for 'B' is 1% greater than that for 'A', find the value of Modulus of Rigidity for the material 'B' in terms of Modulus of Elasticity and Modulus of Rigidity for material 'A'.

[8 marks : 2017]

Solution:

Let E_A, K_A, G_A and E_B, K_B, G_B be the modulus of elasticity, bulk modulus and modulus of rigidity of materials A and B respectively.

Given, $K_A = K_B$ and $E_B = 1.01\, E_A$

We know, $$E = \frac{9KG}{3K + G}$$

or, $3KE + EG = 9\,KG$

or, $3K(3G - E) = EG$

from where, we get $$K = \frac{EG}{3(3G - E)}$$

Hence, $$\frac{E_A G_A}{3(3G_A - E_A)} = \frac{E_B G_B}{3(3G_B - E_B)}$$

$$\therefore \quad E_A G_A (3G_B - E_B) = E_B G_B (3G_A - E_A)$$

or, $3E_A G_A G_B - E_A G_A E_B = 3G_A G_B E_B - E_A E_B G_B$

or, $G_B(3E_A G_A - 3G_A E_B + E_A E_B) = E_A G_A E_B$

$$\therefore \quad G_B = \frac{E_A G_A E_B}{3E_A G_A - 3G_A E_B + E_A E_B}$$

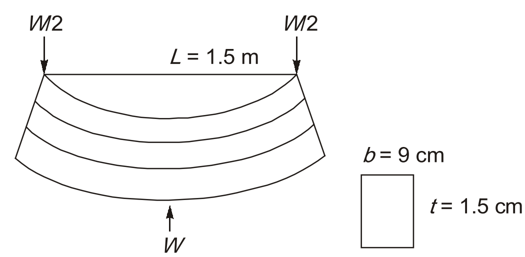
$$\Rightarrow \quad G_B = \frac{1.01 E_A G_A E_A}{3E_A G_A - 3 \times 1.01 E_A G_A + 1.01 E_A \times E_A}$$

or, $$G_B = \frac{1.01 E_A G_A}{3G_A - 3.03 G_A + 1.01 E_A} = \frac{101 E_A G_A}{101 E_A - 3G_A}$$

1.10 A leaf spring of semi-elliptical type has 11 plates each 9 cm wide and 1.5 cm thick. The length of spring is 1.5 m. The plates are made of steel having a proof stress (bending) of 650 MN/m^2. To what radius should the plates be bent initially? From what height can a load of 600 N fall on to centre of the spring, if maximum stress is to be one-half of the proof stress? Take E = 200 GN/m^2.

[8 marks : 2018]

Solution:



$$f_y = 650 \text{ N/mm}^2$$

$$\because \quad \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$I = \frac{bt^3}{12}$$

$$y_{max} = \frac{t}{2}$$

$$E = 200 \text{ GPa} = 200 \times 10^6 \text{ N/mm}^2$$

$$\Rightarrow \quad R = \frac{E \cdot y_{max}}{\sigma_y} = \frac{200 \times 10^3 \times \left(\frac{15}{2}\right)}{650} \text{mm}$$

$$= 2307.69 \text{ mm} = 2.307 \text{ m}$$

$\therefore$ Initial radius, R = 2.307 m

Let the load P = 600 N falls from x height

$$\sigma_{max} = \frac{1}{2} \times \sigma_y = \frac{650}{2} = 325 \text{ N/mm}^2$$

$$P = 600 \text{ N}$$

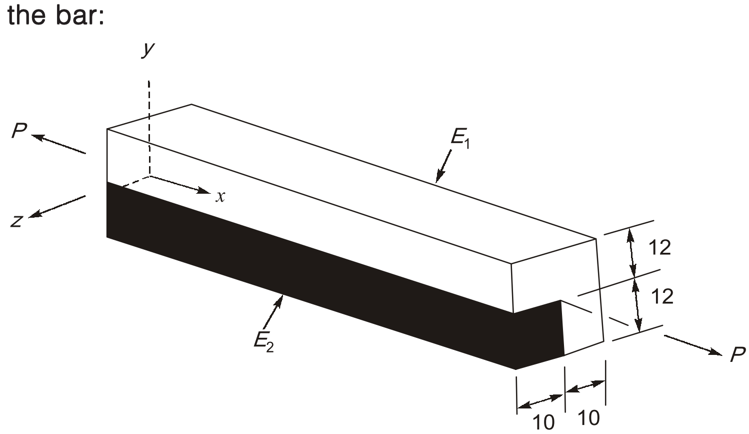
From energy conservation $$Px = \frac{\sigma_y^2}{2E} \times V$$

$$\Rightarrow \quad 600x = \frac{(325 \times 10^6)^2}{2 \times 200 \times 10^9} \times 11 \times 9 \times 1.5 \times 10^{-9} \times 1.5$$

$$\Rightarrow \quad x = 9.80 \text{ m}$$

$\therefore$ Height of fall = 9.80 m

1.11 A composite bar of rectangular cross-section 20 mm × 24 mm is loaded in tension by a force P = 1 kN as shown in fig. below. The shaded part of the bar is made of a material where Young's modulus is E_2 = 210 GPa. The remaining part is made of a material with E_1 = 105 GPa. If the bar is to deflect in the x-direction, determine the stresses in each material and the location of the loading axis relative to the centre of the bar:



[10 Marks : 2019]

Solution:

Given,
$$E_1 = 105 \text{ GPa}; E_2 = 210 \text{ GPa}$$
$$A_1 = 12 \times 20 + 12 \times 10 = 360 \text{ mm}^2$$
$$A_2 = 12 \times 10 = 120 \text{ mm}^2$$

There is deformation in only x-direction.

$\because$ In composite bar, deformation will be same

$\therefore$ for uniform deformation.

$$(\varepsilon_x)_1 = (\varepsilon_x)_2$$
$$\frac{(\sigma_x)_1}{E_1} = \frac{(\sigma_x)_2}{E_2}$$
$$\frac{(\sigma_x)_1}{(\sigma_x)_2} = \frac{E_1}{E_2} = \frac{105}{210} = \frac{1}{2} = 0.5 \quad \text{...(1)}$$

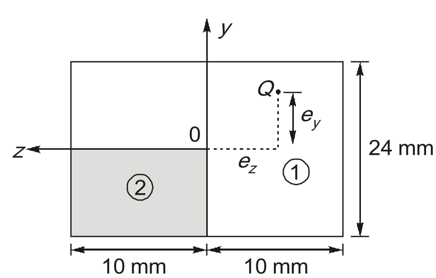
For composite bar,
$$P = P_1 + P_2$$
$$1000 = (\sigma_x)_1 A_1 + (\sigma_x)_2 A_2$$
$$= 0.5(\sigma_x)_2 A_1 + (\sigma_x)_2 A_2$$
$$= 0.5\,(\sigma_x)_2 \times 360 + (\sigma_x)_2 \times 120$$
$$1000 = 300\,(\sigma_x)_2$$
$$(\sigma_x)_2 = \frac{1000}{300} = 3.33 \text{ MPa}$$

from eq. (1)
$$(\sigma_x)_1 = 0.5 \times 3.33 = 1.665 \text{ MPa}$$

Let force is acting at a point Q having y-coordinate e_y and z-coordinate $(-e_z)$

Centroid of section (1)
$$\bar{y}_1 = \frac{120 \times 6 + 240 \times 0}{360} = 2 \text{ mm}$$
$$\bar{z}_1 = \frac{120 \times 5 + 240 \times (-5)}{360} = \frac{-5}{3} \text{mm}$$

Centroid of section (2)
$$\bar{y}_2 = -6 \text{ mm}$$
$$\bar{z}_2 = 5 \text{ mm}$$



By moment equilibrium equation
$$\Sigma(M_0)_y = 0$$
$$1000 \times e_y = P_1\bar{y}_1 + P_2\bar{y}_2$$
$$1000 \times e_y = (\sigma_x)_1 A_1 \cdot \bar{y}_1 + (\sigma_x)_2 \cdot A_2\bar{y}_2$$
$$1000 \times e_y = 1.665 \times 360 \times 2 + 3.333 \times 120 \times (-6)$$
$$e_y = -1.20 \text{ mm}$$

Now,
$$\Sigma(M_0)_z = 0$$
$$1000 \times e_z = P_1\bar{z}_1 + P_2 \cdot \bar{z}_2$$
$$1000 \times e_z = (\sigma_x)_1 A_1\bar{z}_1 + (\sigma_x)_2 A_2 \cdot \bar{z}_2$$
$$1000 \times e_z = 1.665 \times 360 \times \left(-\frac{5}{3}\right) + (3.333 \times 120 \times 5)$$

Hence, location of loading axis relative to the centre of bar is (–1.2 mm, 1 mm)

1.12 In a tension test, a steel rod of gauge length 255 mm and diameter 32 mm was used. The rod during the test was stretched 0.108 mm under a pull of 65 kN. In a torsion test, the same rod was twisted 0.018 radian over a length of 255 mm at the torque of 500×10^3 N-mm. Determine the modulus of elasticity, modulus of rigidity, Poisson's ratio and bulk modulus.

[8 Marks : 2022]

Solution:

Given :

$$\text{Length of rod, } L = 255 \text{ mm}$$
$$\text{Diameter of rod, } d = 32 \text{ mm}$$
$$\text{Pull applied, } P = 65 \times 10^3 \text{ N}$$
$$\text{Elongation in rod, } \Delta = 0.108 \text{ mm}$$

As we know, that when a rod of uniform cross-section is subjected to an axial load,

then,
$$\Delta = \frac{Pl}{AE} \qquad \left[\because A = \frac{\pi}{4}d^2\right]$$

where Δ is axial deformation/elongation and E is modulus of elasticity

Putting values, we get

$$0.108 = \frac{65 \times 10^3 \times 255}{\frac{\pi}{4} \times 32^2 \times E}$$

$$\Rightarrow \quad E = 190827.05 \text{ N/mm}^2$$

Now, when it is subjected to a torsion of 500×10^3 N-mm, angle of twist θ is 0.018 radian.

By torsion formula,

$$\frac{T}{I_P} = \frac{G\theta}{L}$$

where,
$$T = \text{Torsion applied; } I_P = \text{Polar moment of inertia of rod} = \frac{\pi d^4}{32}$$
$$G = \text{Modulus of rigidity; } \theta = \text{Angle of twist; } L = \text{Length of rod}$$

Putting values, we get

$$\frac{500 \times 10^3}{\frac{\pi}{32} \times 32^4} = \frac{G \times 0.018}{255}$$

$$\Rightarrow \quad G = 68807.83 \text{ N/mm}^2$$

As we know
$$E = 2G(1 + \mu) \text{ where } \mu \text{ is Poisson's ratio}$$

Putting values, we get

$$190827.05 = 2 \times 68807.83(1 + \mu)$$

$$\Rightarrow \quad \mu = 0.387$$

Also,
$$E = 3K(1 - 2\mu) \qquad \text{where } K \text{ is bulk modulus}$$

Putting values, we get

$$190827.05 = 3 \times K \times (1 - 2 \times 0.387)$$

$$\Rightarrow \quad K = 281455.83 \text{ N/mm}^2$$

Hence,
$$\text{Modulus of elasticity, } E = 190827.05 \text{ N/mm}^2 = 1.9 \times 10^5 \text{ N/mm}^2$$
$$\text{Modulus of rigidity, } G = 68807.83 \text{ N/mm}^2 \simeq 0.688 \times 10^5 \text{ N/mm}^2$$
$$\text{Bulk modulus of elasticity, } K = 281455.83 \text{ N/mm}^2 \simeq 2.8 \times 10^5 \text{ N/mm}^2$$
$$\text{Poisson's ratio, } \mu = 0.387$$